Spin-Momentum Correlation (Handedness) in the Process of Four Pions Production in the Electron-Positron Collisions

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Abstract

We discuss special type of polarization asymmetry ("handedness") in the process $e^+e^- \to 4\pi$, when one of initial particles is longitudinally polarized. The asymmetry is proportional to the degree of polarization and to the width to mass ratio of rho-meson. It can reach 4-5% in some kinematical region. Both channels $2\pi^+2\pi^-$ and $2\pi^0\pi^+\pi^-$ are considered in the framework of the effective chiral lagrangian with vector mesons, in the energy range ~ 1 GeV. The corresponding total cross sections are also calculated.

1 Introduction

The handedness concept, as a measure of the initial state polarization, was first discussed in papers of O. Nachtman and A. V. Efremov [1, 2]. In particular, it was suggested that the polarization of the initial parton could be established by investigating the characteristics of the corresponding jet [3, 4]. The longitudinal polarization of quark created in e^+e^- annihilation, arises due to the interference between vector and axial amplitudes of the Z-boson intermediate state. However, the correlation between quark polarization and jet handedness is expected to be greatly reduced when averaging over final phase space because of the complicated process of jet fragmentation. So in this case one can expect the effect to be of the order of 2-3%, making its experimental study an elaborate task.

In this paper we consider the similar correlation in a much simpler case of e^+e^- annihilation into four pions at intermediate energies. It allows one to study the most probable mechanism of the handedness generation, namely, the wide resonance imaginary phase contribution. The quantity (handedness)

$$H = \frac{L-R}{L+R} \tag{1}$$

where L,R are the numbers of the left handed and right handed configurations constructed from the two pions 3-momenta and the beam direction, is not zero when electron and

positron beams are longitudinally polarized (it turns out that the effect is also present in the case when only one of the beams is polarized.

We suggest to choosing the same charge sign pions for the $2\pi^+2\pi^-$ channel and arrange them according to their momenta, while any pair of pions can be taken for the $2\pi^0\pi^+\pi^-$ channel.

Note that for $e^+e^- \to 2\pi$ and $e^+e^- \to 3\pi$ reactions the handedness effect is absent $H=0$. This is due to the fact that in these cases we actually have only one amplitude and can therefore measure only the symmetrical part of the initial state spin-density matrix.

In the case of four pions production we have two types of amplitudes which depend differently on the initial polarization and the interference between them just gives the helicity-dependent term. Note that this interference term is due to the nonzero width of the ρ-meson in some intermediate state and the large value of this width suggests that the considerable effect could be expected.

2 General considerations

The main contribution to the $e^+e^- \to 4\pi$ cross section goes from the annihilation channel. Using the vector dominance model, the corresponding matrix element can be presented as

$$M^{e^+e^-\to 4\pi} = \frac{4\pi\alpha m_\rho^2}{s(s-m_\rho^2+im_\rho\Gamma_\rho)}\ \overline{v}(\lambda_+,p_+)\ \gamma_\mu\ u(\lambda_-,p_-)\ J_\mu(\rho^0\to 4\pi), \tag{2}$$

where $s=(p_+ + p_-)^2$, $\lambda_+=-\lambda_-=\pm 1$ are the initial state positron and electron chiralities and $g_{\rho\pi\pi}J_\mu(\rho\to 4\pi)$ is the conserved current:

$$q_\mu\ J^\mu = 0, \quad q = p_+ + p_-, \tag{3}$$

which describes the $\rho\to 4\pi$ transition. Its concrete form is of course model dependent. In the energy region $\sqrt{s}\sim 1$ GeV, which we are going to consider, the effective chiral lagrangian with vector mesons can give a reasonable approximation [6]. We will use the version [7, 8] of such an effective chiral lagrangian, which correctly incorporates a phenomenologically successful vector meson dominance picture [9] and current algebra low energy theorems. For convenience, let us reproduce here its relevant part:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\,\mathrm{Sp}\ (D_\mu\Phi)(D^\mu\Phi) + \frac{1}{2f_\pi^2}\ (\frac{1}{3}-\alpha_k)\mathrm{Sp}\left[\Phi\ (D_\mu\Phi)\ \Phi\ (D^\mu\Phi) - \Phi^2\ (D_\mu\Phi)(D^\mu\Phi)\right] \\ &- \frac{\varepsilon^{\mu\nu\lambda\sigma}}{\pi^2}\left\{\frac{3}{8\sqrt{2}}\frac{g_{\rho\pi\pi}^2}{f_\pi}\ \mathrm{Sp}\left[(\partial_\mu V_\nu)(\partial_\lambda V_\sigma)\Phi\right] + i\frac{g_{\rho\pi\pi}}{4f_\pi^3}(1-3\alpha_k)\ \mathrm{Sp}\left[V_\mu(\partial_\nu\Phi)(\partial_\lambda\Phi)(\partial_\sigma\Phi)\right]\right\} \\ &- \frac{em_\rho^2}{g_{\rho\pi\pi}}A_\mu\rho^\mu - \frac{1}{4}\mathrm{Sp}F_{\mu\nu}^{(V)}F^{(V)\mu\nu},\end{aligned} \tag{4}$$

where $\alpha_k = \dfrac{g_{\rho\pi\pi}^2 f_\pi^2}{m_\rho^2} \simeq 0.55$, $D_\mu\Phi = \partial_\mu\Phi - i\dfrac{g_{\rho\pi\pi}}{\sqrt{2}}[V_\mu,\Phi]$, $F_{\mu\nu}^{(V)} = \partial_\mu V_\nu - \partial_\nu V_\mu - i\dfrac{g_{\rho\pi\pi}}{\sqrt{2}}[V_\mu,V_\nu]$, $f_\pi\simeq 93$ MeV and Φ, V_μ are the conventional SU(3) matrices for pseudoscalar and vector meson fields.

From (2) we get

$$| M |^2 = \frac{(4\pi\alpha m_\rho^2)^2}{s^2\left[(s-m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2\right]}\ L_{\mu\nu}\ J^\mu\ J^{\nu\dagger}, \tag{5}$$

where $L_{\mu\nu}$ lepton tensor has only transversal to beam components in the center of mass system:

$$L_{\mu\nu} = \frac{1}{4}\ \mathrm{Sp}\ \hat{p}_-(1+\lambda_-\gamma_5)\gamma_\mu\hat{p}_+(1-\lambda_+\gamma_5)\gamma_\nu = \frac{s}{2}\ \left[(1-\lambda_+\lambda_-)\delta^\perp_{\mu\nu} + i(\lambda_- - \lambda_+)\varepsilon^\perp_{\mu\nu}\right],$$
$$\delta_{\mu\nu} = diag(0,1,1,0),\ \varepsilon^\perp_{\mu\nu} = \varepsilon_{03\mu\nu},\ \varepsilon_{0123} = 1. \tag{6}$$

Due to the presence of the nonzero imaginary part of the $\rho \to 4\pi$ amplitude, $J_\mu J_\nu^\dagger$ tensor has an antisymmetrical part:

$$J_\mu J_\nu^\dagger = (a+ib)_\mu\ (a-ib)_\nu = a_\mu a_\nu + b_\mu b_\nu + a_\mu a_\nu + i(b_\mu a_\nu - a_\mu b_\nu). \tag{7}$$

As a result we obtain for the cross section:

$$d\sigma^{e^+e^-\to 4\pi} = \frac{(\alpha m_\rho^2)^2\mathcal{F}}{2^6\pi^6 s^2\left[(s-m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2)\right]}\ \prod_{i=1}^{4}\frac{d\vec{q}_i}{2E_i}\ \delta^4(q-\sum_{i=1}^{4}q_i),$$
$$\mathcal{F} = (1-\lambda_+\lambda_-)\ \left(a_x^2 + a_y^2 + b_x^2 + b_y^2 + 2(\lambda_- - \lambda_+)(\vec{a}\times\vec{b})_z\right), \tag{8}$$

where it is assumed that z-axis coincides with the $\vec{p}_-$-direction.

Performing the phase space integration, one may obtain $\sigma_{L,R}$ in the form

$$\sigma_{L,R} = \frac{1}{2}(1-\lambda_+\lambda_-)\sigma_0 \pm \frac{1}{2}(\lambda_- - \lambda_+)\frac{\Gamma_\rho}{m_\rho}\sigma_1. \tag{9}$$

In fact σ_0 is an unpolarized cross section and σ_1 is related to the spin-momentum correlation (handedness):

$$H = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{\Gamma_\rho}{m_\rho}\frac{\lambda_- - \lambda_+}{1-\lambda_+\lambda_-}\frac{\sigma_1}{\sigma_0} \tag{10}$$

3 Four charged pions production

The types of Feynman diagrams for transition $\rho \to 2\pi^+2\pi^-$:

$$e^+(p_+) + e^-(p_-) \to \pi^+(q_1) + \pi^+(q_2) + \pi^-(q_3) + \pi^-(q_4) \tag{11}$$

are shown in Fig. 1. The corresponding current has a form:

$$J_\mu^{\rho^0\to 2\pi^+2\pi^-} = (\frac{1}{3}-\alpha_k)\frac{1}{f_\pi^2}\left[6(q_1+q_2-q_3-q_4)_\mu + (6q_3.q_4 + 2m^2)\left(\frac{(q-2q_1)_\mu}{(q-q_1)^2-m^2}\right.\right.$$
$$\left.\left. + \frac{(q-2q_2)_\mu}{(q-q_2)^2-m^2}\right) - (6q_1.q_2 + 2m^2)\left(\frac{(q-2q_3)_\mu}{(q-q_3)^2-m^2} + \frac{(q-2q_4)_\mu}{(q-q_4)^2-m^2}\right)\right]$$
$$+2(1+P_{12})(1+P_{34})\frac{g^2_{\rho\pi\pi}((q_2+q_4)^2 - m_\rho^2 - im_\rho\Gamma_\rho)}{((q_2+q_4)^2 - m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2}$$
$$\times\left[(q_4-q_2)_\mu + \frac{q_1.(q_2-q_4)}{(q-q_3)^2-m^2}(q-2q_3)_\mu + \frac{q_3.(q_2-q_4)}{(q-q_1)^2-m^2}(q-2q_1)_\mu\right], \tag{12}$$

where $m^2 = m_\pi^2 = q_i^2$. P_{12} and P_{34} operators stand for the interchange of the corresponding identical mesons momenta.

Consider now the equally charged pions (π^+ for example), arranged according to the magnitude of their momenta (say, more energetic particle defines an x-axis direction), and let them together with the beam axis (for definiteness $\vec{p}_-$) form a left or right configurations. The numbers of the left and right repers will not in general coincide, if the initial state is characterized by some nonzero average longitudinal polarization. The corresponding assymmetry (handedness) is given by (10). Using a standard covariant phase-space calculations [10], (8) and (9) can be cast in the following form

$$\sigma_{0,1} = \frac{(\alpha m_\rho^2)^2}{2^7 \pi^6 s^2} \frac{R_{0,1}}{\left[(s - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2)\right]}, \tag{13}$$

where

$$R_0 = \frac{\pi^2}{24s} \int\limits_{s_1^-}^{s_1^+} ds_1 \int\limits_{s_2^-}^{s_2^+} ds_2 \int\limits_{u_1^-}^{u_1^+} \frac{du_1}{\sqrt{\lambda(s, s_2, s_2')}} \int\limits_{u_2^-}^{u_2^+} du_2 \int\limits_{-1}^{1} \frac{d\zeta}{\sqrt{1-\zeta^2}} |\vec{J}|^2 \tag{14}$$

(the expressions for the integration limits, as well as some details of calculations are given in the appendix). Assuming that $\vec{J}$ from (12) is presented as

$$\vec{J} = D_1 \vec{q}_1 + D_2 \vec{q}_2 + D_3 \vec{q}_3, \tag{15}$$

R_1 is given by a similar expression

$$R_1 = \frac{\pi^2}{8s} \int\limits_{s_1^-}^{s_1^+} ds_1 \int\limits_{s_2^-}^{s_2^+} ds_2 \int\limits_{u_1^-}^{u_1^+} du_1 \frac{\theta(u_1 - s_1)}{\sqrt{\lambda(s, s_2, s_2')}} \int\limits_{u_2^-}^{u_2^+} du_2 \int\limits_{-1}^{1} \frac{d\zeta}{\sqrt{1-\zeta^2}} \sqrt{\frac{\Delta_3(q, q_1, q_2)}{s}} \left(\frac{m_\rho}{\Gamma_\rho} f_1\right), \tag{16}$$

where

$$f_1 = \frac{i}{2}(D_1 D_2^\star - D_2 D_1^\star) \tag{17}$$

and

$$\begin{aligned} \Delta_3(q, q_1, q_2) &= \begin{vmatrix} q{\cdot}q & q{\cdot}q_1 & q{\cdot}q_2 \\ q_1{\cdot}q & q_1{\cdot}q_1 & q_1{\cdot}q_2 \\ q_2{\cdot}q & q_2{\cdot}q_1 & q_2{\cdot}q_2 \end{vmatrix} \\ &= \begin{vmatrix} s & \frac{1}{2}(s + m^2 - s_1) & \frac{1}{2}(s + m^2 - u_1) \\ \frac{1}{2}(s + m^2 - s_1) & m^2 & \frac{1}{2}(s + s_2 - s_1 - u_1) \\ \frac{1}{2}(s + m^2 - u_1) & \frac{1}{2}(s + s_2 - s_1 - u_1) & m^2 \end{vmatrix} \end{aligned} \tag{18}$$

$\theta(u_1 - s_1)$ in (16) is equivalent to $\theta(E_1 - E_2)$ and expresses an arrangement of identical pions according to their energy.

The results of the numerical calculations are presented in Fig. 2. The unpolarized total cross section σ_0 is also shown in Fig. 3 together with the experimental data [11].

4 $2\pi^0\pi^+\pi^-$– channel

For the process

$$e^+(p_+) + e^-(p_-) \to \pi^+(q_+) + \pi^-(q_-) + \pi^0(q_1) + \pi^0(q_2) \tag{19}$$

some additional Feynman diagrams with the vertices from the anomalous part of chiral Lagrangian are essential. The types of relevant diagrams are drawn in Fig. 1. The corresponding current J_μ can be presented as a sum of three terms, each representing a gauge invariant subset of diagrams:

$$J_\mu^{\rho\to 2\pi_0\pi_+\pi_-} = J_\mu^{(1)} + J_\mu^{(2)} + J_\mu^{(3)}. \tag{20}$$

Diagrams of type a,b of Fig. 1 give:

$$J_\mu^{(1)} = (\frac{1}{3} - \alpha_k)\frac{1}{f_\pi^2}(6q_1.q_2 + 2m_{\pi^0}^2)\ \left[\frac{(q-2q_-)_\mu}{(q-q_-)^2 - m_{\pi^\pm}^2} - \frac{(q-2q_+)_\mu}{(q-q_+)^2 - m_{\pi^\pm}^2}\right]. \tag{21}$$

The second piece arises from diagrams of type c,d,e of Fig. 1 and has the form

$$\begin{aligned}
J_\mu^{(2)} &= -g_{\rho\pi\pi}^2\,(1+P_{12})\left\{-\frac{1}{r_+r_-}\,[2(q_+-q_1)_\mu q.(q_--q_2) - 2(q_--q_2)_\mu q.(q_+-q_1)\right.\\
&+ (q_2+q_--q_1-q_+)_\mu(q_+-q_1).(q_--q_2)]\\
&+ \frac{1}{r_+}\left[(q_+-q_1)_\mu - 2q_2.(q_+-q_1)\frac{(q-2q_-)_\mu}{(q-q_-)^2-m_\pi^2}\right]\\
&- \left.\frac{1}{r_-}\left[(q_--q_2)_\mu - 2q_1.(q_--q_2)\frac{(q-2q_+)_\mu}{(q-q_+)^2-m_\pi^2}\right]\right\}, \qquad (22)\\
r_+ &= (q_++q_1)^2 - m_\rho^2 + im_\rho\Gamma_\rho; \quad r_- = (q_-+q_2)^2 - m_\rho^2 + im_\rho\Gamma_\rho.
\end{aligned}$$

Finally, the third part of current is determined by two diagrams of type f of Fig. 1 with the ω-meson intermediate state:

$$J_\mu^{(3)} = \frac{3g_{\rho\pi\pi}}{8\pi^2 f_\pi}\,(1+P_{12})\,P_\mu\,\frac{F_1}{r_1}, \tag{23}$$

where

$$P_\mu = q_1.q_2(q_{+\mu}q.q_- - q_{-\mu}q.q_+) + q_-.q_2(q_{1\mu}q.q_+ - q_{+\mu}q.q_1) + q_+.q_2(q_{-\mu}q.q_1 - q_{1\mu}q.q_-), \tag{24}$$

and

$$\begin{aligned}
r_1 &= (q-q_2)^2 - m_\omega^2 + im_\omega\Gamma_\omega,\\
F_1 &= \frac{3g_{\rho\pi\pi}}{4\pi^2 f_\pi^3}\left[1 - 3\alpha_k - \alpha_k\left(\frac{m_\rho^2}{r_{+-}} + \frac{m_\rho^2}{r_{+1}} + \frac{m_\rho^2}{r_{-1}}\right)\right],\\
r_{+-} &= (q_++q_-)^2 - m_\rho^2 + im_\rho\Gamma_\rho,\\
r_{+1} &= (q_++q_1)^2 - m_\rho^2 + im_\rho\Gamma_\rho,\\
r_{-1} &= (q_1+q_-)^2 - m_\rho^2 + im_\rho\Gamma_\rho. \qquad (25)
\end{aligned}$$

The handedness value in the case when two π^0's are taken to define a reper is less a 1At last, in Fig. 4 we draw the calculated total unpolarized cross section compared to the experimental data from [11].

The known experimental data for $\sqrt{s} < 1$ GeV [11] are in resonable agreement with our calculation of total cross section for $2\pi^0\pi^+\pi^-$ channel. In calculations, we have taken into account the dependence of the ρ-meson width on energy. The situation is worse for $\sqrt{s} > 1$ GeV. For $\sqrt{s} = 1.3$ GeV the experimental cross section exceeds about one order of magnitude the ones obtained above (8) (See Fig. 4). Presumably, the difference arises mainly from the influence of the ρ-meson radial exitation — ρ' (1450) resonance. Let us now introduce an additional factor $R(s)$ in the cross section $d\sigma(s) \to d\sigma(s)R(s)$,

$$R(s) = \left|\frac{m_\rho^2}{s - m_\rho^2 + im_\rho\Gamma_\rho}\right|^{-2} \cdot \left|\frac{m_\rho^2}{s - m_\rho^2 + im_\rho\Gamma_\rho} + \frac{m_{\rho'}^2 e^{i\varphi}}{s - m_{\rho'}^2 + im_{\rho'}\Gamma_{\rho'}}\right|^2, \qquad (26)$$

which takes into account the ρ'-meson contribution. From Figs. 3,5 we see that it works in the useful direction. Note in conclusion, that the value of handedness (10) will not be changed after the replacement $d\sigma \to Rd\sigma$.

As for a_1-meson contribution, for low energy region $\sqrt{s} < 1$ GeV it is effectively taken into account via the effective coupling constant in the lagrangian. This obviously becomes incorrect when $\sqrt{s} > 1.3$ GeV, where 3π-invariant mass can reach such values that the Breit-Wigner character of a_1-meson intermediate state propagator is essential. It's the resons why the above given formulas can not be applied in the $\sqrt{s} > 1.3$ GeV region.

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References

[1] O. Nachtman, Nucl. Phys. **B127** (1977) 314.

[2] A. V. Efremov, Sov. J. Nucl. Phys. **28** (1978) 83.

[3] A. V. Efremov, L. Mankiewiez, N. A. Törnqvist, Phys. Lett **284B** (1992) 394.

[4] R. H. Dalitz, G. R. Goldstein, R. Marshall, Z. Phys. **C42**, (1989) 441.

[5] A. V. Efremov, L. Mankiewiez, N. A. Törnqvist, Preprint CERN-TH.6574192, 1992.

[6] O. Kaymakcalan, S. Rajeev, J. Schechter, Phys. Rev. **D30** (1984) 594.

[7] Y. Brihaye, N. K. Pak, P. Rossi, Nucl. Phys. **B254** (1985) 71; Phys. Let. **164B** (1985) 111.

[8] E. Kuraev and Z. K. Silagadze, Phys. Lett. **292B** (1992) 377; Z. K. Silagadze, Preprint BUDKERINP 92-61, Novosibirsk, 1992.

[9] J. J. Sakurai, Currents and Mesons, University of Chicago Press, Chicago, 1969; P. J. O'Donnell, Rev. Mod. Phys. **53** (1981) 673.

[10] R. Kumar, Phys. Rev. **185** (1969) 1865; E. Byckling, K. Kajantie, Particle Kinematics, Wiley, London, 1973.

[11] S. I. Dolinsky et al., Phys. Rep. **202** (1991) 99.

A Appendix. Covariant phase-space calculations

Let us consider

$$R_4 = \int |\mathcal{M}|^2 \delta(q - \sum_{j=1}^{4} q_j) \prod_{i=1}^{4} \frac{d\vec{q}}{2E_i}. \tag{A.1}$$

If we introduce Kumar's invariant variables

$$s_1 = (q-q_1)^2, s_2 = (q-q_1-q_2)^2, u_1 = (q-q_2)^2,\ u_2 = (q-q_3)^2, t_2 = (q-q_2-q_3)^2 \tag{A.2}$$

(A.1) can be recasted in the form [10] (assuming that $|M|^2$ is rotational invariant):

$$R_4 = \frac{\pi^2}{8M^2} \int_{s_1^-}^{s_1^+} ds_1 \int_{s_2^-}^{s_2^+} ds_2 \int_{u_1^-}^{u_1^+} du_1 \int_{u_2^-}^{u_2^+} du_2 \int_{-1}^{1} \frac{d\zeta}{\sqrt{1-\zeta^2}} \frac{|\mathcal{M}|^2}{\sqrt{\lambda(s, s_2, s_2')}} \tag{A.3}$$

where $s_2' = s_2 + s + m_1^2 + m_2^2 - u_1 - s_1$ and $\arccos\zeta$ is an angle between $(\vec{q}_2, \vec{q}_1 + \vec{q}_2)$ and $(\vec{q}_3, \vec{q}_1 + \vec{q}_2)$ planes. $\lambda(x,y,z) = (x+y+z)^2 - 4xy$ is a conventional triangle function. t_2 and ζ are related by

$$\begin{aligned} t_2 &= m_3^2 + u_1 - \frac{(s+u_1-m_2)^2(s+m_3^2-u_2)}{2s} - \frac{\{\lambda(s,m_2^2,u_1)\lambda(s,m_3^2,u_2)\}^{1/2}}{2s} \\ &\times (\xi\eta - \zeta\sqrt{(1-\xi^2)(1-\eta^2)}), \end{aligned} \tag{A.4}$$

$\arccos\xi$ and $\arccos\eta$ being angles, respectively, between $\vec{q}_2$ and $\vec{q}_1 + \vec{q}_2$ vectors, and $\vec{q}_3$ and $\vec{q}_1 + \vec{q}_2$ vectors. They can be expressed by invariant variables (A.2) as follows [10]:

$$\begin{aligned} \xi &= \frac{\lambda(s,s_2,s_2') + \lambda(s,m_2^2,u_1) - \lambda(s,m_1^2,s_1)}{2\{\lambda(s,s_2,s_2')\lambda(s,m_2^2,u_1)\}^{1/2}} \\ \eta &= \frac{\lambda(s,m_4^2,s_3') - \lambda(s,s_2,s_2') - \lambda(s,m_3^2,u_2)}{\{\lambda(s,s_2,s_2')\lambda(s,m_3^2,u_2)\}^{1/2}}, \end{aligned} \tag{A.5}$$

where $s_3' = 2s + \sum_{i=1}^{4} m_i^2 - s_1 - u_1 - u_2$. The limits of integration for s-type variables are

$$s_1^- = (m_2+m_3+m_4)^2,\ s_1^+ = (\sqrt{s} - m_1)^2,\ s_2^- = (m_3+m_4)^2,\ s_2^+ = (\sqrt{s_1} - m_2)^2. \tag{A.6}$$

While the limits for u-type variables are defined from $|\xi| < 1, |\eta| < 1$ and look like

$$\begin{aligned} u_1^\pm &= s + m_2^2 - \frac{(s_1+m_2^2-s_2)(s+s_1-m_1^2)}{2s_1} \pm \frac{\{\lambda(s_1,m_2^2,s_2)\lambda(s,s_1,m_1^2)\}^{1/2}}{2s_1} \\ u_2^\pm &= s + m_3^2 - \frac{(s_2+m_3^2-m_4^2)(s+s_2-s_2')}{2s_2} \pm \frac{\{\lambda(s_2,m_3^2,m_4^2)\lambda(s,s_2,s_2')\}^{1/2}}{2s_2} \end{aligned} \tag{A.7}$$

If we use (A.3) and note that for σ_0, $|\mathcal{M}|^2 = |J_x|^2 + |J_y|^2$ can be replaced by $\frac{2}{3}|\vec{J}|^2$ and $u_1 > s_1$ condition, which is assumed when calculating $\sigma_{L,R}$, can be omitted and replaced by a factor $\frac{1}{2}$, we recover (14) formula.

Dealing with σ_1 more care is needed when integrating over $\vec{q}_1$ and $\vec{q}_2$ angular variables. It is assumed in (A.3) that $|\mathcal{M}|^2$ doesn't depend from three of them and so these integrations give $8\pi^2$. This is no longer true in the case of σ_1, because now $|\mathcal{M}|^2 = 2(\vec{a} \times \vec{b})_z$. After integrating over $d\vec{q}_3$, this can be replaced by $|\mathcal{M}|^2 = f_1(\vec{q}_1 \times \vec{q}_2)_z$. Let us choose the following system for $d\vec{q}_2$ integration: z-axis is along $\vec{q}_1$ and $\vec{p}_-$ vector lies in the x, z-plane, than $(\vec{q}_1 \times \vec{q}_2) \cdot \hat{\vec{p}}_- = -|\vec{q}_1||\vec{q}_2| \sin\theta_1 \sin\theta_2 \sin\varphi_2$. Left or right reper means $(\vec{q}_1 \times \vec{q}_2) \cdot \hat{\vec{p}}_- > 0$ or $(\vec{q}_1 \times \vec{q}_2) \cdot \hat{\vec{p}}_- < 0$ and so $\pi \leq \varphi_2 \leq 2\pi$ for left configuration and $0 \leq \varphi_2 \leq \pi$ for right one. Therefore the integration over $d\varphi_2$ gives $\pm 2|\vec{q}_1||\vec{q}_2| \sin\theta_1 \sin\theta_2$. The integration over $d\Omega_1 = \sin\theta_1 d\theta_1 d\varphi_1$ now gives a factor π^2. So the net effect of these integrations is the change $|\mathcal{M}|^2 \to \pm 2\pi^2 |\vec{p}_1||\vec{p}_2| \sin\theta_2$. It can be checked [10] that $\sin\theta_2 = \sin\theta_{12} = \dfrac{1}{|\vec{p}_1||\vec{p}_2|}\sqrt{\dfrac{\Delta_3(q, q_1, q_2)}{s}}$, and so we recover the result for σ_1 cited in the text.

Figure captions

Fig. 1. The Feynman diagrams for the process $e^+e^- \to 4\pi$.

Fig. 2. The handedness value in the case $e^+ + e^- \to \pi^+ + \pi^+ + \pi^- + \pi^-$.

Fig. 3. The unpolarized total cross section σ_0 for the process $e^+ + e^- \to \pi^+ + \pi^+ + \pi^- + \pi^-$. The experimental data are taken from Ref.[11]. Dashed line — ρ' meson is added according to (26) with $\varphi = 180^o$.

Fig. 4. The unpolarized total cross section σ_0 for the process $e^+ + e^- \to \pi^0 + \pi^0 + \pi^+ + \pi^-$. The experimental data are taken from Ref.[11]. Dashed line — ρ' meson is added according to (26) with $\varphi = 180^o$.